Exam 1 Practice Problems

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1. Let $f(x) = \frac{1-\sqrt{x}}{1-x}$, $g(x) = \frac{\sin x}{x}$ and $h(x) = \frac{(2+x)^2-4}{x}$. Find the desired limits.
   (a) $\lim_{x\to 1} f(x)$.
   (b) $\lim_{x\to 0} g \cdot h(x)$.
   (c) $\lim_{x\to 2} \frac{f}{g}(x)$.
   (d) $\lim_{x\to 1} \frac{h}{f}(x)$.
   (e) $\lim_{x\to \pi}(f \cdot h - g \cdot f)(x)$.

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2. Find explicitly the (best possible) continuous extension of f, g and h from problem 1.

3. Use the intermediate value theorem.

   (a) Find an interval on which $y = x^3 - x - 1$ has a zero.

   (b) Show that the equation $\sqrt{2x+5} = 4 - x^2$ has a solution.

   (c) Show that the equation $\cos x = x^2$ has a solution.

   (d) Show that the graph of the equation $x^3 - 3x$ crosses the line $y = 1$.

   (e*) Let f be a continuous function on the interval $[0, 1]$. Suppose that $0 \leq f(x) \leq 1$ for every $x \in [0, 1]$. Show that there must be a number $c \in [0, 1]$ such that $f(c) = c$. (c is called a fixed point of f)

4. Do the following problems involving limits at infinity.

   (a) Find $\lim_{x\to\infty} x\frac{\sin 4/x}{2}$.

   (b) Find all asymptotes of $y = \left(\frac{x^2+x-1}{8x^2-3}\right)^{1/3}$.

   (c) Find all asymptotes of $h(t) = \frac{t^3+7t^2-2}{t^2-t+1}$.

   (d) Consider $g(x) = \left(\frac{x^2}{2} - \frac{1}{x}\right)$. Find the limits of $g(x)$ as $x \to 0^+$, $x \to 0-$, $x \to \sqrt[3]{2}$ and $x \to -1$.